**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

:= Mean service time = 45 min.

Std service time = 8 min.

Remaining time = 1 hr – 10 min = 50min.

Z score = 50-45/8 = 0.625

Z table = 0.73237

Probability of not meeting commitement = 1-0.73237= 0.2676

Option B is correct.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

:= *μ* = 38

*σ* =6

P(x>44)= 1-p(x<=44)

P(x<=44)=p(z<=44)= (x-*μ)* /*σ*

=(44-38)/6

=1

Z table value =0.8413

P(z>44) =1-p(z<=44)

=1-0.8413

=0.1587

=15.87%

Probability that the employers between 38 &44= P(x>=38) -P(x<=44)

P(x<=44) = 84.13%

P(x>=38)=P(z>=38)= (x-*μ)*/*σ*

=(38-38)/6

=0

Z table value = 0.5

=50%

P(x>=38) -P(x<=44) = 84.13 -50

=34.13%

more employes are older than 44 years.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

:= mean age = 38

Std age = 6

Target age = 30

Z score = target age – mean age/std age

Z score = 30-38/6 = -1.33.

Z table =0.09176

No of employes under the age 30 = 400\*0.09176 = 36.76.

The given statement is true,so employess under 30 age would except to attract about 36 employes.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

:= 2X1 =2( μ, σ2 ) = two times the mean of X1 and four times the varience of X1 because of square present in it.

X1 + X2 = sum of mean *μ*1​+*μ*2​ and sum of varience 1σ2 + 2σ2 .

In reference both are normally distributed.but mean and varience differs based on specific linear transformation.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

:= mean = 100

std = 20

z\_value = 2.576

a = mean - z\_value \* std\_dev

b = mean + z\_value \* std\_dev

a = 100-2.576\*20 =48.48

b = 100+2.576\*20=151.52.

so option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

:= mean of total profit = mean of prifit1 + profit2

= 5 + 7 = 12

Mean of rupees = 12\*45 = 540

Std of total profit = std of profit 1 + profit 2

= sqrt(9+16)=sqrt25 = 5

Std in rupees = 5\*45=225

Mean = 540,std = 225 ,confidence interval = 0.95,z\_score = 1.96

Margin of error = Z\_score \* std

=1.96\*225 = 441

Lower\_bound = mean -margin of error

=540-441 = 99

Upper\_bound = mean + margin of error

=540 + 441 = 981

1. Specify the 5th percentile of profit (in Rupees) for the company

:= from scipy.stats import norm

mean = 540

std = 225

percentile = 5

percentile\_value = norm.ppf(percentile / 100, loc=mean, scale=std)

print(f'The 5th percentile of profit is: {percentile\_value}')

the 5th percentile of profit = 169.90

1. Which of the two divisions has a larger probability of making a loss in a given year?

:= 2 nd division is making loss in given year of 0.3773%.